OCR

Friday 6 June 2014 – Afternoon

A2 GCE MATHEMATICS

4730/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4730/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A particle P of mass 0.3 kg is moving on a smooth horizontal surface with speed $0.8\,\mathrm{m\,s^{-1}}$ when it is struck by a horizontal impulse. The magnitude of the impulse is 0.6 N s.

(i) (a) Find the greatest possible speed of P after the impulse acts.

(b) Find the least possible speed of P after the impulse acts. **[3]**

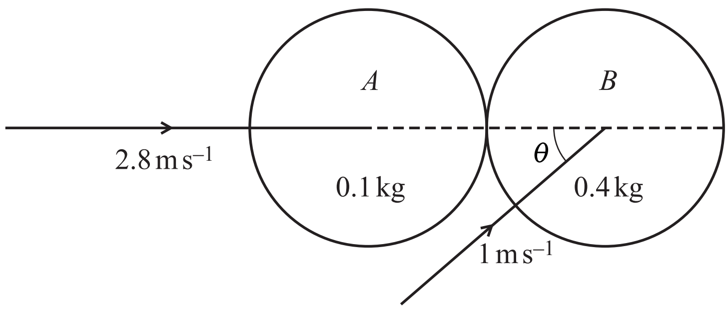
(ii) In fact the speed of P after the impulse acts is $2.5\,\mathrm{m\,s^{-1}}$. Find the angle the impulse makes with the original direction of travel of P and draw a sketch to make this direction clear. **[4]**

2 One end of a light elastic string, of natural length 0.6 m and modulus of elasticity 30 N, is attached to a fixed point O. A particle P of weight 48 N is attached to the other end of the string. P is released from rest at a point d m vertically below O. Subsequently P just reaches O.

(i) Find d. **[4]**

(ii) Find the magnitude and direction of the acceleration of P when it has travelled 1.3 m from its point of release. **[4]**

3



Two uniform smooth spheres A and B of equal radius are moving on a horizontal surface when they collide. A has mass 0.1 kg and B has mass 0.4 kg. Immediately before the collision A is moving with speed $2.8\,\mathrm{m\,s^{-1}}$ along the line of centres, and B is moving with speed $1\,\mathrm{m\,s^{-1}}$ at an angle θ to the line of centres, where $\cos\theta = 0.8$ (see diagram). Immediately after the collision A is stationary. Find

(i) the coefficient of restitution between A and B, **[5]**

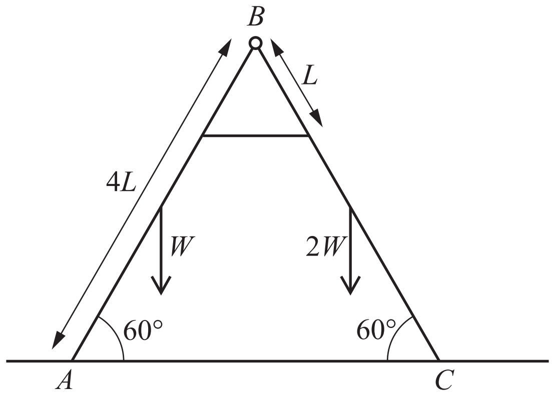
(ii) the angle turned through by the direction of motion of B as a result of the collision. **[4]**

4 A particle P of mass 0.4 kg is projected horizontally with speed $2\,\mathrm{m\,s^{-1}}$ from a fixed point O on a smooth horizontal surface. At time t s after projection P is x m from O and is moving away from O with speed $v\,\mathrm{m\,s^{-1}}$. There is a force of magnitude $1.6v^2$ N resisting the motion of P.

(i) Find an expression for $\frac{\mathrm{d}v}{\mathrm{d}x}$ in terms of v, and hence show that $v = 2\mathrm{e}^{-4x}$. **[5]**

(ii) Find the distance travelled by P in the 0.5 seconds after it leaves O. **[5]**

5



Two uniform rods AB and BC, each of length $4L$, are freely jointed at B, and rest in a vertical plane with A and C on a smooth horizontal surface. The weight of AB is W and the weight of BC is $2W$. The rods are joined by a horizontal light inextensible string fixed to each rod at a point distance L from B, so that each rod is inclined at an angle of 60° to the horizontal (see diagram).

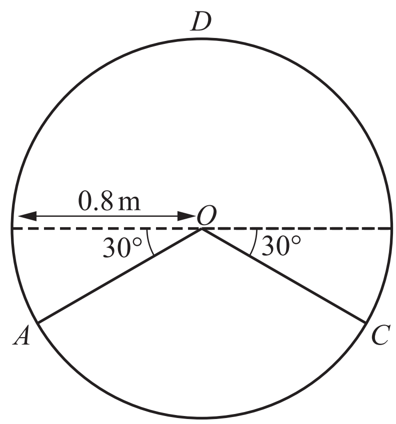
(i) By considering the equilibrium of the whole body, show that the force acting on BC at C is $1.75W$ and find the force acting on AB at A. **[4]**

(ii) Find the tension in the string in terms of W. **[4]**

(iii) Find the horizontal and vertical components of the force acting on AB at B, and state the direction of the component in each case. **[3]**

Question 6 begins on page 4.

6

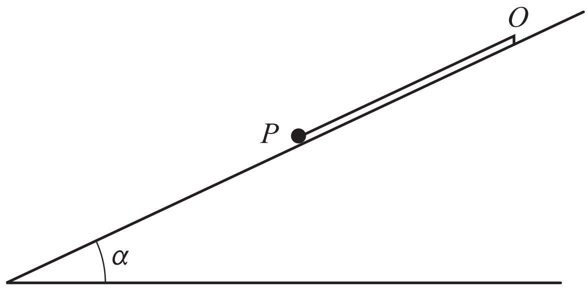


A hollow cylinder is fixed with its axis horizontal. O is the centre of a vertical cross-section of the cylinder and D is the highest point on the cross-section. A and C are points on the circumference of the cross-section such that AO and CO are both inclined at an angle of 30° below the horizontal diameter through O. The inner surface of the cylinder is smooth and has radius 0.8 m (see diagram). A particle P, of mass m kg, and a particle Q, of mass $5m$ kg, are simultaneously released from rest from A and C, respectively, inside the cylinder. P and Q collide; the coefficient of restitution between them is 0.95.

(i) Show that, immediately after the collision, P moves with speed $6.3\,\mathrm{m\,s^{-1}}$, and find the speed and direction of motion of Q. **[8]**

(ii) Find, in terms of m, an expression for the normal reaction acting on P when it subsequently passes through D. **[6]**

7



One end of a light elastic string, of natural length 0.3 m, is attached to a fixed point O on a smooth plane that is inclined at an angle α to the horizontal, where $\sin\alpha = 0.2$. A particle P of mass m kg is attached to the other end of the string. The string lies along a line of greatest slope of the plane and has modulus of elasticity $2.45m$ N (see diagram).

(i) Show that in the equilibrium position the extension of the string is 0.24 m. **[2]**

P is given a velocity of $0.3\,\mathrm{m\,s^{-1}}$ down the plane from the equilibrium position.

(ii) Show that P performs simple harmonic motion with period 2.20 s (correct to 3 significant figures), and find the amplitude of the motion. **[6]**

(iii) Find the distance of P from O and the velocity of P at the instant 1.5 seconds after P is set in motion. **[5]**

Answer			Marks	Guidance	
1	(i)	realising impulse must be in same direction as velocity, or opposite max speed 2.8 (m s^{-1}) min speed 1.2 (m s^{-1})	M1 A1 A1 **[3]**	 0.8 +/− 0.6/0.3 − 1.2 is wrong	 various methods
	(ii)	Impulse momentum diagram $\cos\theta = \frac{0.6^2 + 0.24^2 - 0.75^2}{2 \times 0.6 \times 0.24}$ $\theta = 120°$ (2.098 rad) angle shown correctly	M1 A1 M1 A1 **[4]**	Triangle with sides labelled 0.24, 0.6 and 0.75 or 0.8, 2 and 2.5 accept 59.8° (1.04 rad) consistent with their θ; dep M1A1M1	Allow M1 if positions wrong. Diagram must be correct. $v_x = 0.8 + 2\cos\theta$ M1 either $v_y = 2\sin\theta$ and correct diag A1 both Square, add, giving 1.61 = 3.2cosθ M1 120.(21)...A1
2	(i)	By energy $\frac{30(d-0.6)^2}{2 \times 0.6} = 48 \times d$ $25d^2 - 78d + 9 = 0$ or $30d^2 - 93.6d + 10.8 = 0$ (d =) 3 (m)	M1* A1 *M1 A1 **[4]**	Attempt at elastic energy get 3 term quadratic and attempt to solve ignore d = 0.12, unless given as answer	Allow M1 for $\frac{30y^2}{(2)\times 0.6} = kd$ $\frac{30x^2}{2 \times 0.6} = 48(x + 0.6)$ allow 1 slip or $25x^2 - 48x - 28.8 = 0$ (x =) 2.4 leading to (d =) 3
	(ii)	Use $F = ma$ $48 - \frac{30 \times (3 - 0.6 - 1.3)}{0.6} = (\pm)\frac{48}{g}a$ (a =) (+/−) 1.43 upwards	M1 A1ft A1 A1 **[4]**	 ft their '3' 1.4291666 depends on a being right	allow missing g, allow 1.3 or 0.6 to be omitted Using energy: $a = v\frac{dv}{dx} = \frac{g}{48}(50x - 72)$ M1A1

		Answer	Marks	Guidance	
3	(i)	Using conservation of momentum along loc 0.1 x 2.8 + 0.4 x 1x0.8 = 0.4 x b Using NEL $b - 0 = -e(1\text{x}0.8 - 2.8)$ $e = 0.75$	M1 A1 M1 A1 A1 **[5]**	3 (or 4) terms, correct dimensions Vel diff after = e x vel diff before	Allow sign errors, (sin/cos) may see $b = 1.5$ Allow $\pm e$
	(ii)	$b(perp) = 0.6$ $\tan\beta = \frac{b(perp)}{\text{their } 1.5}$, angle turned through is 36.9° − β = 15.1° (0.262 rad)	B1 M1* *M1 A1 **[4]**	 β = 21.8°; ft 1.5 from (i) Must be 36.9° − their β (soi)	May be on diagram 21.8014...(0.381 rad) 36.86989 15.068 scB1 for 165° after B1M1
4	(i)	Use $F = mv\frac{dv}{dx}$ $-4v = \frac{dv}{dx}$ $-4x = \ln v + c$ $0 = \ln 2 + c$ $\ln\frac{v}{2} = -4x$ $v = 2e^{-4x}$	M1 A1 M1 M1 A1 **[5]**	 expression for $\frac{dv}{dx}$ required get (+/–) $Ax = \ln v + c$ valid attempt to find c need a step leading to given answer AG	Allow sign error, missing m or g inc
	(ii)	$e^{4x}\,dx = 2\,dt$ $\frac{1}{4}e^{4x} = 2t + c$ $\frac{1}{4} = 0 + c$ $e^{4x} = 4(1 + \frac{1}{4})$ $x = \frac{1}{4}\ln 5$	M1* A1 *M1 *M1 A1 **[5]**	Write v as $\frac{dx}{dt}$ and separate variables must have c or use limits valid attempt to find c or subst limits find x when $t = 0.5$ - need to remove exp; allow even if no c Accept 0.402(359…)	$dv/4v^2 = -dt$ $\frac{1}{v} = 4t + \frac{1}{2}$ $\frac{dx}{dt} = \frac{2}{8t+1}$ OR t = 0.5 gives v = 0.4 $x = \frac{1}{4}\ln(8t+1) + c$ OR $-4x = \ln 0.2$ $x = \frac{1}{4}\ln 5$
5	(i)	Take moments about A for whole body Wx2Lcos60° + 2Wx6Lcos60° = Rx8Lcos60° $R = 1.75W$ $S = 1.25W$	M1 A1 A1 B1 **[4]**	Correct 3 terms needed; dim correct cos60° may be omitted at least 1 correct step to show given answer	Allow sign errors, W/2W, cos/sin, R is reaction at C S is reaction at A For less efficient methods, M1 can only be earned when equation with one unknown, R, is reached.

Answer			Marks	Guidance	
	(ii)	Take moments about *B* for equil of *BC* *T*x*L*sin60° + 2*W*x2*L*cos60° = 1.75*W*x4*L*cos60° solve to get $T = \sqrt{3}W$	M1* A1 *M1 A1 **[4]**	Correct 3 resolved terms needed; dim correct; or for *BA* *T*x*L*sin60° + *W*x2*L*cos60° = 1.25*W*x4*L*cos60° accept *T* = 1.73*W*	allow sign errors, *W*/2*W*, cos/sin,
	(iii)	Resolve vertically for *AB* *Y* + 1.25*W* – *W* = 0 *Y* = 0.25*W*, downwards $X = \sqrt{3}W$ to left	 M1 A1CAO B1ft **[3]**	 direction must be clear direction must be clear	 Weight and normal term must be for same rod
6	(i)	$\frac{1}{2}mv^2 = mg \times 0.8(1 - \sin 30°)$ $v = 2.8$ m s^{-1} Speed of P and Q equal Use conservation of momentum 5*m*x2.8 – *m*x2.8 =5*mq* + *mp* Use of NEL *p* – *q* = –0.95(– 2.8 – 2.8) *p* = 6.3 m s^{-1} *q* = 0.98 m s^{-1} *Q* moves to left	M1 A1 B1ft B1ft M1 A1ft A1 A1 **[8]**	Or with ‘5*m*’ if for *Q* soi Ft on velocity Ft on velocity supporting work required forAG direction must be clear	allow *g* missing for M1. Might see $v^2 = 0.8g$ *p* is vel of *P*, *q* is vel of *Q*, both to left Allow ± *e*
	(ii)	By energy for *P* at top $\frac{1}{2}m6.3^2 = \frac{1}{2}mv^2 + mg \times 1.6$ $v^2 = 8.33$ Use *F* = *ma* at top $mg + R = m \times \frac{8.33}{0.8}$ *R* = 0.6125*m*	M1 A1 A1 M1 A1ft A1CAO **[6]**	must have 3 terms Soi must have 3 terms their v^2 Or 49*m*/80	allow *g* missing, sign error allow *g* missing, sign error

Answer			Marks	Guidance	
7	(i)	$mg \times 0.2 = \frac{2.45m \times e}{0.3}$ $e = 0.24$	M1 A1 **[2]**	 No errors; must show all numbers	allow sin/cos, wrong sign, missing g
	(ii)	Use $F = ma$ down slope $mgsin\alpha - \frac{2.45m(x-0.3)}{0.3} = m\ddot{x}$ $\ddot{x} = -\frac{49}{6}(x - 0.54)$ SHM (about $x = 0.54$) $\omega = 7/\sqrt{6}$ (2.8577) $T = 2.20$ $a = 0.105$ m (0.1049795)	M1 A1 A1 B1 B1CAO B1ft **[6]**	3 terms needed oe Accept 2.45/0.3 for ω^2 Dep M1A1. Must be in correct form, and ω^2 in simplified form Soi AG Need to see $2\pi/\omega$ oe ft their ω $\frac{3\sqrt{6}}{70}$	Allow sign error, sin/cos, missing g or m Could use x in place of $x - 0.3$, leading to $\ddot{x} = -\frac{49}{6}(x - 0.24)$(about $x = 0.24$) Or $x + 0.24$ in place of $x - 0.3$ leading to $\ddot{x} = -\frac{49}{6}x$ (about $x = 0$) May see $\omega^2 = 8\frac{1}{6}$ 2.1986568... NB Can find a by energy, leading to ω and T
	(iii)	Use of SHM eqn for distance $x = -0.0956(227...)$ Dist from O is 0.444(377...) (m) Use of SHM equation for velocity $v = -0.124$ (−0.123949...)	M1 A1ft A1CAO M1 A1 **[5]**	$x = a\sin\omega t$ Their a $v = a\omega\cos\omega t$ must be clear velocity is towards O	Allow M1 for $x = a\cos\omega t$ Or -0.9553 or -0.09577 Allow M1 for $v = -a\omega\sin\omega t$ if consistent with x eqn for sin/cos, a, ω Use of $v^2 = \omega^2(a^2 - x^2)$ will not gain A1 unless direction is established